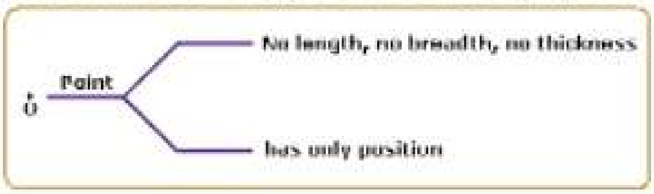
Lines and Angles

Geometrical Concepts

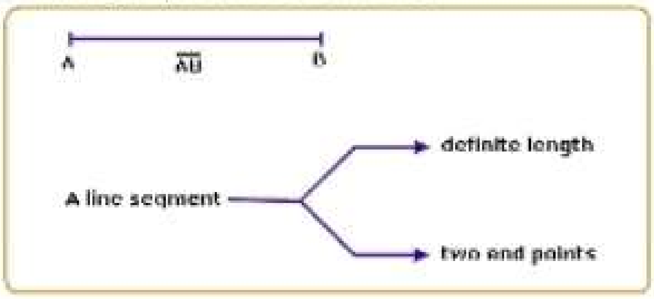
Point

It is an exact location. It is a fine dot which has neither length nor breadth nor thickness but has position i.e., it has no magnitude. It is denoted by capital letters A, B, C, O etc.



Line Segment

The straight path joining two points A and B is called a line segment $\overline{AB}$. It has end points and a definite length. (no breadth or thickness)



Ray

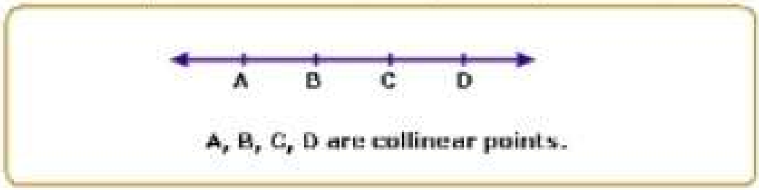
A line segment which can be extended in only one direction is called a ray.

Line

When a line segment is extended indefinitely in both directions it forms a line.

Collinear Points

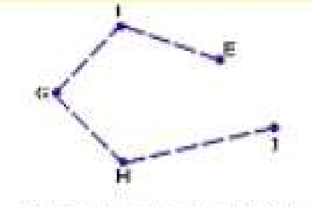
If two or more points lie on the same line, then they are called collinear points.



Non-collinear Points

Points which do not lie on the same line are called non-collinear points.

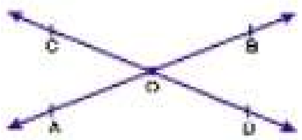
Example: E, F, G, H, I



E, F, G, H, I are non-collinear points.

Intersecting Lines

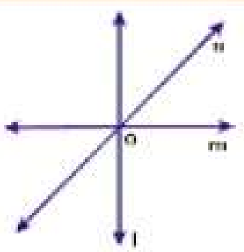
Two lines having a common point are called intersecting lines. The common point is known as the point of intersection.



O is the point of intersection

Concurrent Lines

If two or more lines intersect at the same point, then they are known as concurrent lines.



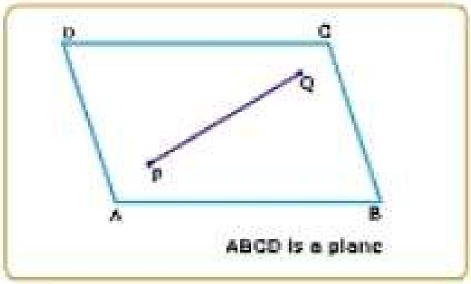
l, m, n are concurrent lines

Plane

A plane is a surface such that every point of the line joining any two points on it, lies on it.

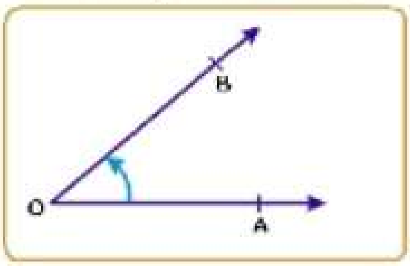
Example:

Surface of a smooth wall, surface of a paper.



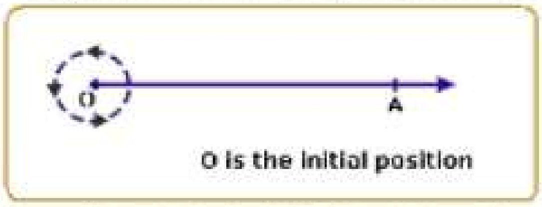
Angles

When two straight lines meet at a point they form an angle.



It is represented as $\angle AOB$ or $A\hat{O}B$.

- $\overline{OA}$ and $\overline{OB}$ are called the arms of $\angle AOB$.
- The point at which the arms meet (O) is known as the vertex of the angle.
- The amount of turning from one arm (OA) to other (OB) is called the measure of the angle ($\angle AOB$) and written as $m\angle AOB$.
- An angle is measured in degrees, minutes and seconds.
- If a ray rotates about the starting initial position, in anticlockwise direction, comes back to its original position after 1 complete revolution then it has rotated through 360°.



$\Rightarrow$ 1 complete rotation is divided into 360 equal parts. Each part is 1°.

Each part (1°) is divided into 60 equal parts, each part measuring one minute, written as $1'$.

$1'$ is divided into 60 equal parts, each part measuring 1 second, written as $1''$.

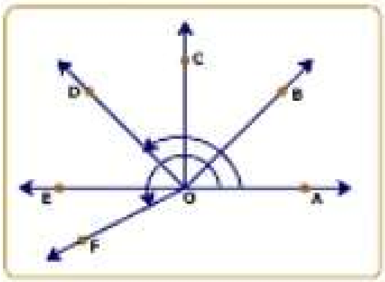
Degrees ------> minutes --------> seconds

$1^\circ = 60'$

$1' = 60''$

Recall that the union of two rays forms an angle.

In the figure, observe the different types of angles:

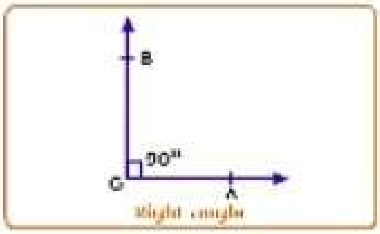


- $A\hat{O}B$ is an acute angle ($0^0 < A\hat{O}B < 90^0$)
- $A\hat{O}C$ is a right angle (an angle equal to 90^0)
- $A\hat{O}D$ is an obtuse angle ($90^0 < A\hat{O}D < 180^0$)
- $A\hat{O}E$ is a straight angle (an angle equal to 180^0)
- $A\hat{O}F$ (measured in anticlock wise direction) is a reflex angle

  ($180^0 < A\hat{O}F < 360^0$)

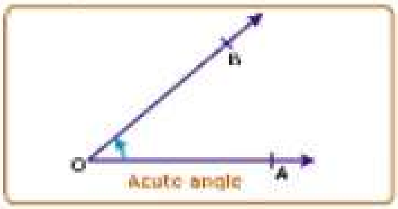
Right Angle

An angle whose measure is 90^0 is called a right angle.



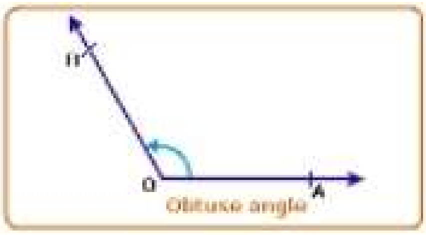
Acute Angle

An angle whose measure is less than one right angle (i.e., less than 90^0), is called an acute angle.



Obtuse Angle

An angle whose measure is more than one right angle and less than two right angles (i.e., less than 180^0 and more than 90^0) is called an obtuse angle.



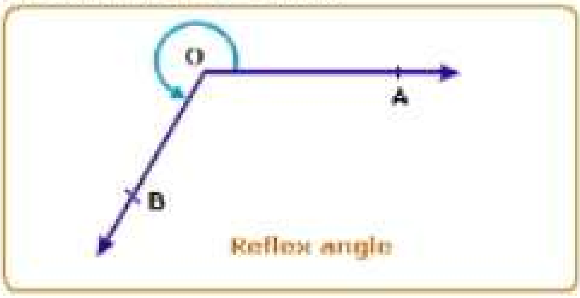
Straight Angle

An angle whose measure is 180° is called a straight angle.

Reflex Angle

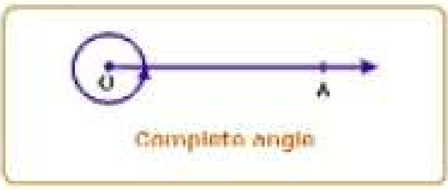
An angle whose measure is more than 180° and less than 360° is called a reflex angle.

It is written as ref. ∠AOB.



Complete Angle

An angle whose measure is 360° is called a complete angle.

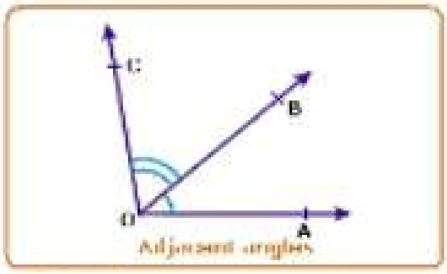


Equal Angles

Two angles are said to be equal, if they have the same measure.

Adjacent angles

Two angles having a common vertex and a common arm, such that the other arms of these angles are on opposite sides of the common arm, are called adjacent angles.



- O is the common vertex.
- $A\hat{O}B$ and $B\hat{O}C$ are adjacent angles.
- Arm BO separates the two angles.

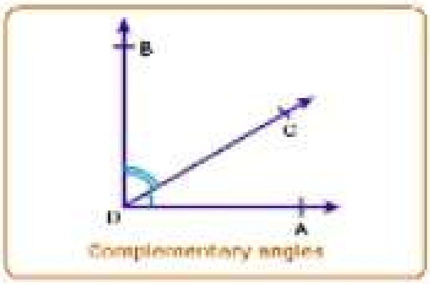
Complementary Angles

If the sum of the two angles is one right angle (i.e., 90°), they are called complementary angles.

If the measure of $A\hat{O}C = a^{0}$, $C\hat{O}B = b^{0}$, then $a^{0} + b^{0} = 90^{0}$.

Therefore $A\hat{O}C$ and $C\hat{O}B$ are complementary angles.

$A\hat{O}C$ is complement of $C\hat{O}B$.



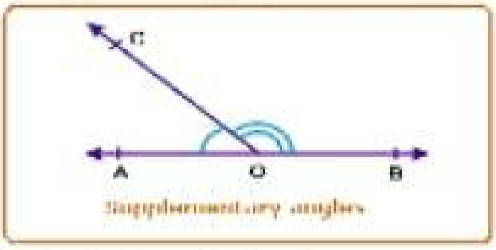
Supplementary Angles

Two angles are said to be supplementary, if the sum of their measures is 180°.

Example:

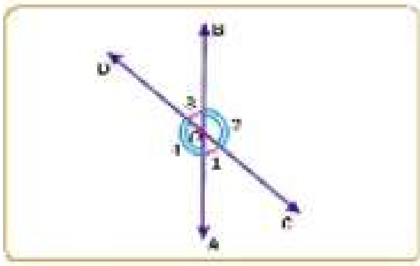
Angles measuring 130° and 50° are supplementary angles.

Two supplementary angles are the supplement of each other.



Vertically Opposite Angles

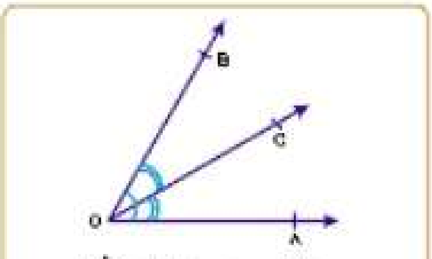
When two straight lines intersect each other at a point, the pairs of opposite angles so formed are called vertically opposite angles.



Angles $\angle 1$ and $\angle 3$ and angles $\angle 2$ and $\angle 4$ are vertically opposite angles. Vertically opposite angles are always equal.

Bisector of an Angle

If a ray or a straight line passing through the vertex of that angle, divides the angle into two angles of equal measurement, then that line is known as the Bisector of that angle.



$\overrightarrow{OC}$ is the bisector of $A\hat{O}B$

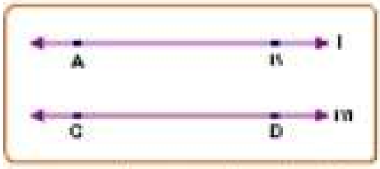
$$B\hat{O}C = C\hat{O}A$$

and $B\hat{O}C + C\hat{O}A = A\hat{O}B$

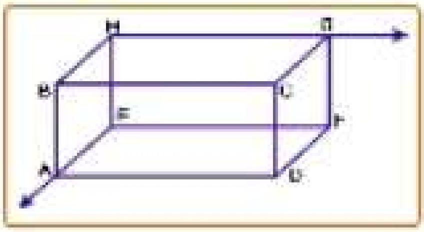
and $A\hat{O}B = 2B\hat{O}C = 2C\hat{O}A$

Parallel Lines

Two lines are parallel if they are coplanar and they do not intersect each other even if they are extended on either side.

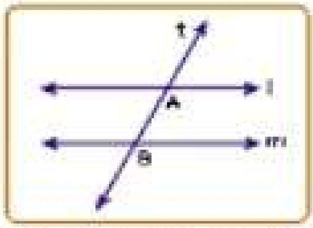


However, there are lines that do not intersect and yet are not parallel. They are skew lines. Skew lines are lines that are not coplanar and do not intersect. AE and HG are skew lines.



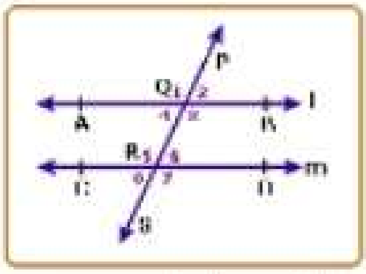
Transversal

Observe the three lines 'l', 'm' and 't'.



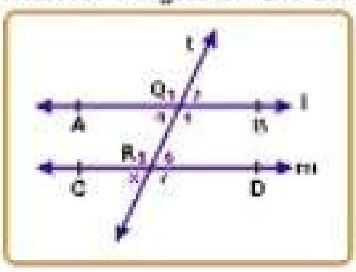
In the diagram 'l' and 'm' are two parallel lines. 't' intersects 'l' at two distinct points 'A' and 'B' and 'm' at 'C' and 'D'. Line t is called a transversal.
A transversal is a line that intersects (or cuts) two or more parallel lines at distinct points.

Angles Formed by a Transversal



In the diagram $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$are two parallel lines. PQRS is a transversal intersecting $\overleftrightarrow{AB}$ at Q and $\overleftrightarrow{CD}$ at R. Eight angles are formed, they are numbered from 1 to 8. By virtue of their locations, some of the angles can be paired together. The paired angles are given special names (apart from adjacent angles and vertical angles).

Interior Angles on the Same side of the Transversal



In the diagram, $A\hat{Q}R(\angle 4)$ and $Q\hat{R}C(\angle 5)$ and $B\hat{Q}R(\angle 3)$ and $Q\hat{R}D(\angle 6)$ form two pairs of interior angles on the same side of the transversal.

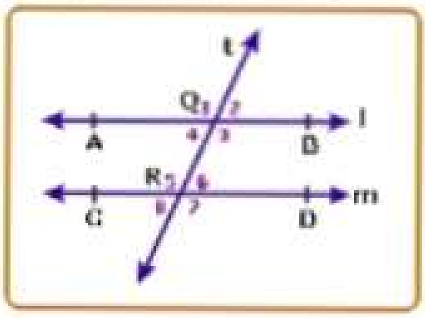
Alternate Angles

A pair of angles are said to be alternate angles if

(i) both are interior angles

(ii) they are on the opposite sides of the transversal and

(iii) they are not adjacent angles.



Alternate angles are sometimes also called alternate interior angles.

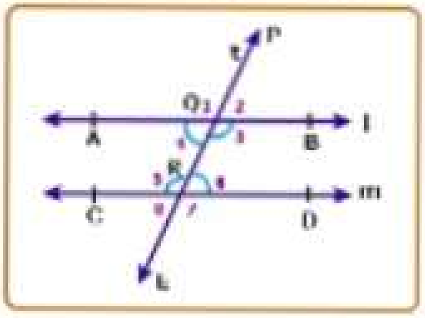
In the diagram,

- $A\hat{Q}R$ and $Q\hat{R}D$ ($\angle 4$ and $\angle 6$)
- $B\hat{Q}R$ and $Q\hat{R}C$ ($\angle 3$ and $\angle 5$)

} are two pairs of alternate angles.

Corresponding Angles

- A pair of angles are said to be corresponding angles if
- One is an interior angle and the other is an exterior angle
- They are on the same side of the transversal and
- They are not adjacent angles.



The four pairs of corresponding angles are given below.

- $A\hat{Q}P$ and $C\hat{R}Q$ ($\angle 1$ and $\angle 5$)
- $A\hat{Q}R$ and $C\hat{R}E$ ($\angle 4$ and $\angle 8$)
- $B\hat{Q}R$ and $D\hat{R}E$ ($\angle 3$ and $\angle 7$)

PAGE NO.:
DATE: / /

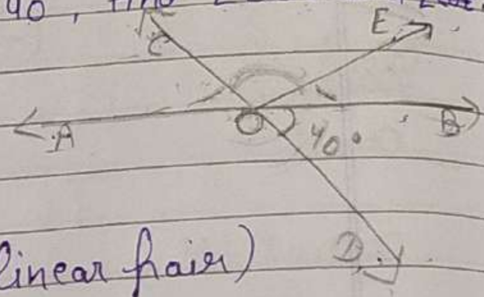
Chapter - 6

Lines and Angles

Ex 6.1

Theorem 6.1 = If two lines intersect each other, then the vertically ophosite angles are equal.

Q1. In fig. 6.13 lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and $\angle COE$. and reflex $\angle COE$.



Sol: On line AB

$\angle AOC + \angle BOE + \angle COE = 180°$ (linear fair)

$70° + \angle COE = 180°$

$\angle COE = 180 - 70 = 110°$

Reflex $\angle COE = 360 - 110°$

$= 250°$

On line CD

$\angle COE + \angle BOE + \angle BOD = 180°$ (linear fair)

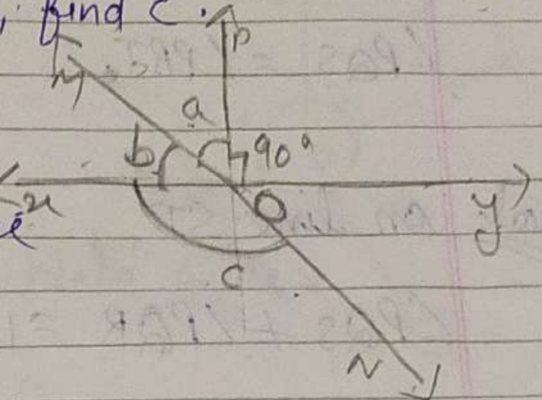
$110° + \angle BOE + 40° = 180°$

$\angle BOE + 150° = 180°$

$\angle BOE = 180 - 150$

$\angle BOE = 30°$

Q2. In fig 6.14 lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.



Sol:
Common ratio of a and b is x

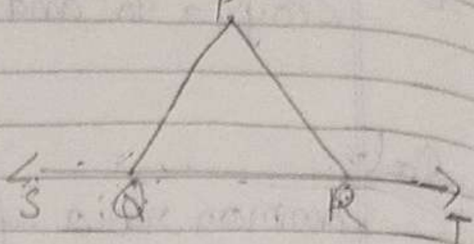
$a = 2x$, $b = 3x$

On line xy
$\angle POY + a + b = 180°$ (linear pair)
$90° + 2x + 3x = 180°$
$90° + 5x = 180°$
$5x = 180 - 90°$
$5x = 90°$
$x = \frac{90}{5} = 18°$

$a = 2x = 2 \times 18 = 36°$
$b = 3x = 3 \times 18 = 54°$

On line MN
$b + c = 180°$ (linear pair)
$54 + c = 180°$
$c = 180 - 54°$
$c = 126°$

Q3. In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Sol: On line ST

$\angle PQS + \angle PQR = 180^\circ$ (linear pair)

$\angle PQR = 180^\circ - \angle PQS$ ——— (1)

$\angle PRT + \angle PRQ = 180^\circ$ (linear pair)

$\angle PRQ = 180^\circ - \angle PRT$ ——— (2)

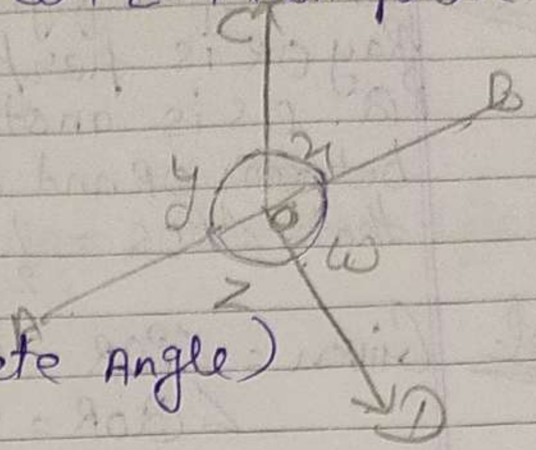
Given $= \angle PQR = \angle PRQ$

$180^\circ - \angle PQS = 180^\circ - \angle PRT$

$+\angle PQS = \cancel{180^\circ} + \angle PRT \cancel{-180^\circ}$

$\angle PQS = \angle PRT$ (H.P)

Q4. In the given fig. If $x+y = w+z$, then prove that AOB is a line.



Sol. Given $= x+y = w+z$

$x+y+w+z = 360°$ (Complete Angle)

$x+y+x+y = 360°$

$2x+2y = 360°$

$2(x+y) = 360°$

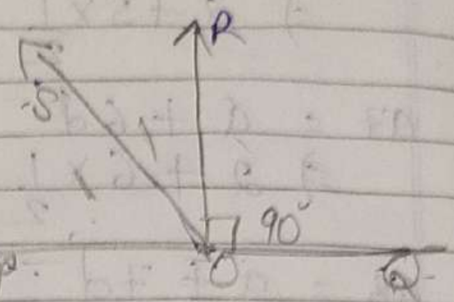
$x+y = \frac{360°}{2}$ $180°$

x and y makes a linear pair so. AOB is a line.

PAGE NO.:
DATE: / /

Day 18 Aug 2020 Ex. 6.1

Q5. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



Sol. Given: $\angle POR = 90°$ [$OR \perp PQ$]

$\angle POS + \angle ROS = 90°$

$\angle ROS = 90° - \angle POS$ — (i)

$\angle QOR = 90°$ [$OR \perp PQ$]

$\angle QOS - \angle ROS = 90°$

$\angle ROS = \angle QOS - 90°$ — (ii)

Add. eq (i) & (ii)

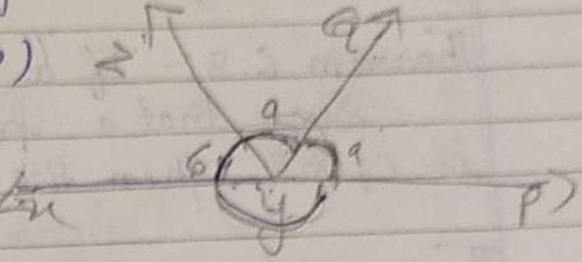
$2\angle ROS = 90° - \angle POS + \angle QOS - 90°$

$2\angle ROS = (\angle QOS - \angle POS)$

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$ H.P.

Q6. It is given that $\angle xyz = 64°$ and xy is produced to point P. Draw a fig. from the given information. If ray yQ bisect $\angle zyP$, Find $\angle xyQ$ and reflex $\angle QyP$.

Sol. $\angle xyz + a + a = 180°$ (l.p)



$$64° + 2a = 180°$$
$$2a = 180 - 64°$$
$$2a = 116$$
$$a = \frac{116}{2} = 58°$$

$\angle xyQ = \cancel{64 + a}$
$64 + 58 = 122°$

Reflex $\angle QyP = 360° - 58°$
302 Ans

PAGE NO. :
DATE : / /

Day 19 Aug 2020 Ex-6.2

Theorem 6.2:- If a transversal intersect two parallel line, then each pair of alternate interior angles is equal.

Theorem 6.3:- If a transversal intersect two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

Theorem 6.4:- If a transversal intersect two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Theorem 6.5:- If a transversal intersect two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

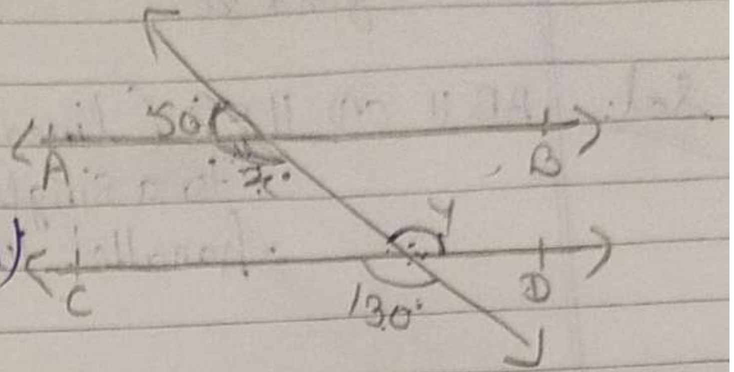
Theorem 6.6:- Lines which are parallel to the same line are parallel to each other.

Theorem 6.7:- The sum of the angles of a triangle is 180°.

Theorem 6.8:- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Ex-6.2

Q1. In fig 6.28, find the Value of x and y then Show that $AB \parallel CD$.



Sol: In this fig.

$50 + x = 180°$ (linear pair)

$x = 180 - 50°$

$x = 130°$

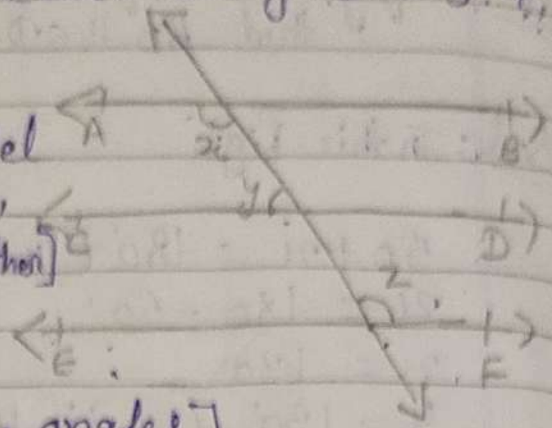
$y = 130°$ [vertically opp. angles]

$x = y = 130°$

x and y are equal and Alternate Interior angles

$\therefore AB \parallel CD$ [Converse of Alternate Angles].

Q2. In fig 6.29, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.



Sol. $AB \parallel CD \parallel EF$ (lines parallel to a single line, parallel to each other)

$x = z$ [Alternate Interior angles]

$x + y = 180°$ [Co - interior angles]

$y + z = 180°$

$3a + 7a = 180°$

$10a = 180°$

$a = \frac{18\cancel{0}}{1\cancel{0}} = 18$

$y = 3a$

$= 3 \times 18 = 54$

$z = 7a$

$= 7 \times 18 = 126°$

$x = z$

$x = 126°$ Ans

Q5 In fig. 6.30, if AB ∥ CD, EF ⊥ CD and ∠GED = 126
find ∠AGE, ∠GEF and ∠FGE.

Sol. $\angle AGE = \angle GED$
(Alternate Interior angles)

$\angle AGE = 126°$

$\angle GEF + \angle FED = \angle GED$
$\angle GEF + 90° = 126°$
$\angle GEF = 126 - 90$
$\angle GEF = 36$

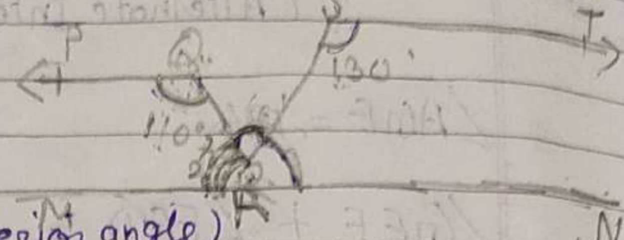
$\angle AGE + \angle FGE = 180°$ (Linear pair)
$126° + \angle FGE = 180°$
$\angle FGE = 180 - 126°$
$\angle FGE = 54°$

$\angle AGE = 126°$
$\angle GEF = 36°$
$\angle FGE = 54$ A

PAGE NO.:
DATE: / /

Q4. In fig. 6.31, if PQ ∥ ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

[Hint: Draw a line parallel to ST through Point R].

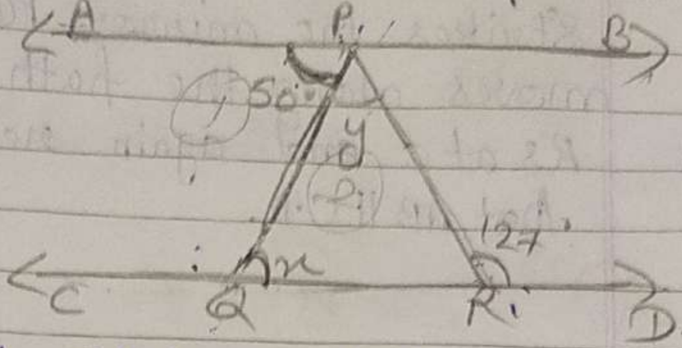


Sol. $110° + x = 180°$ (Co - Interior angle)
$x = 180 - 110°$
$x = 70°$
$\angle MRQ = 70°$

$\angle RST + \angle NRS = 180°$ (Co - Interior angle)
$130° + \angle NRS = 180°$
$\angle NRS = 180 - 130° = 50°$
$\angle MRQ + \angle QRS + \angle NRS = 180°$ [Linear pair]
$70° + \angle QRS + 50 = 180°$
$\angle QRS + 120 = 180°$
$\angle QRS = 180 - 120° = 60°$

PAGE NO. :
DATE : / /

Q5. In fig 6.32, if $AB \parallel CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



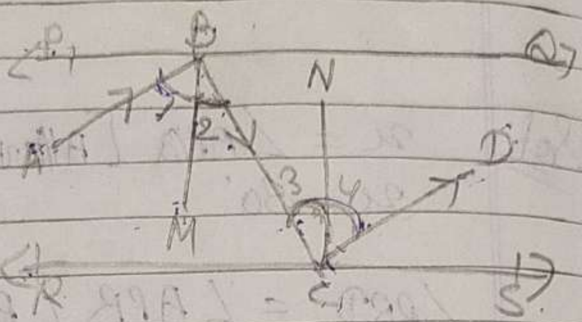
Sol. $x = \angle APQ$ [Alternate Interior angles]
$x = 50°$

$\angle PRD = \angle APR$ [Alternate Interior Angle]
$127 = \angle APQ + y$
$127 = 50 + y$
$50 + y = 127$
$y = 127 - 50°$
$y = 77$

$x = 50$, $y = 77$ Ans.

DATE: / /

Q6. In fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB ∥ CD.



Sol. Given ⇒ PQ ∥ RS, AB, BC, CD are rays
To prove = AB ∥ CD
Const. ⇒ BM ⊥ PQ, CN ⊥ RS
Proof = BM ∥ CN [Perpendiculars on parallel are also parallel]

$\angle 1 = \angle 2$ [Law of Reflection] — (1)
$\angle 3 = \angle 4$

$\angle 2 = \angle 3$ (Alternate Interior Angle] — (2)

$\angle 1 = \angle 2 = \angle 3 = \angle 4$ [By eq (1) & (2)]

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

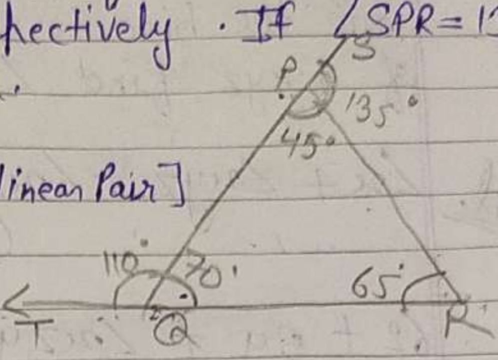
$\angle ABC = \angle BCD$

These are Alternate Interior Angles.

∴ AB ∥ CD (H.P)

Day - 20 Aug 2020 Ex - 6.3

Q1 - In fig. 6.39, Sides QP and RQ of ΔPQR are produced to point S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.



Sol - $\angle SPR + \angle QPR = 180°$ [~~liear~~linear Pair]

$135 + \angle QPR = 180°$

$\angle QPR = 180 - 135°$

$\angle QPR = 45°$

$\angle PQT + \angle PQR = 180°$ [linear pair]

$110 + \angle PQR = 180°$

$\angle PQR = 180 - 110$

$\angle PQR = 70°$

$\angle PQR + \angle QPR + \angle PRQ = 180°$ (Angle sum pro. of Δ)

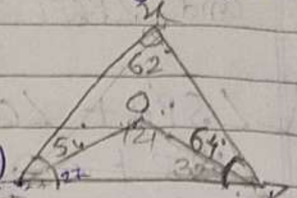
$70° + 45 + \angle PRQ = 180°$

$115° + \angle PRQ = 180°$

$\angle PRQ = 180 - 115°$

$\angle PRQ = 65°$

Q2. In fig 6.40 $\angle x = 62°$, $\angle xyz = 54°$. If yo and zo are the bisectors of $\angle xyz$ and $\angle xzy$ respectively of $\triangle xyz$ find $\angle ozy$ and $\angle yoz$.



Sol. $\angle x + \angle xyz + \angle xzy = 180°$
(Angle sum pro. of $\triangle$)

$62 + 54 + \angle xzy = 180°$

$116° + \angle xzy = 180°$

$\angle xzy = 180 - 116 = 64°$

$\angle ozy = \frac{\angle xzy}{2}$

$\angle ozy = \frac{64}{2} = 32°$

$\angle oyz = \frac{\angle xyz}{2}$

$\angle oyz = \frac{54}{2} = 27°$

In $\triangle oyz$

$\angle oyz + \angle ozy + \angle yoz = 180°$ (Angle sum prop. of $\triangle$)

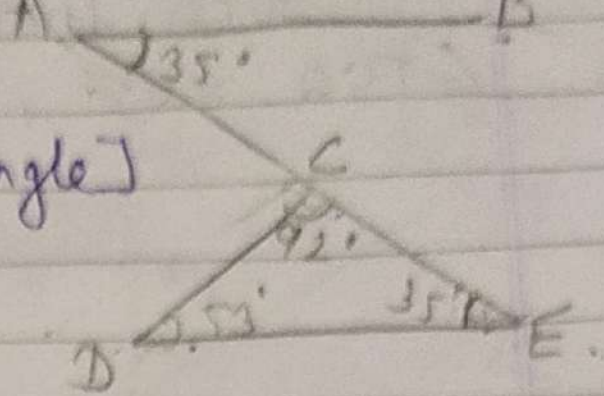
$27 + 32 + \angle yoz = 180°$

$59 + \angle yoz = 180$

$\angle yoz = 180 - 59$

$\angle yoz = 121°$

Q3. In fig. 6.41 If $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.



Sol. $\angle BAC = \angle CED$ [Alternate Interior Angle]

$\angle BAC = 35 = \angle CED$

$\angle CED = 35^\circ$

In ΔCDE

$\angle CDE + \angle CED + \angle DCE = 180^\circ$ [Angle sum prop. of Δ]

$53 + 35 + \angle DCE = 180^\circ$

$88 + \angle DCE = 180^\circ$

$\angle DCE = 180 - 88^\circ$

$\angle DCE = 92^\circ$

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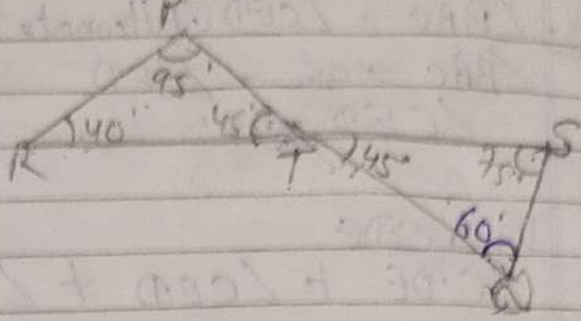
Q4. In fig. 6.42 if lines PQ and RS intersect at point T, Such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.



Sol. In $\triangle PRT$

$\angle RPT + \angle PRT + \angle PTR = 180^\circ$ [Angle Sum proph. of $\triangle$]

$95^\circ + 40^\circ + \angle PTR = 180^\circ$

$135^\circ + \angle PTR = 180^\circ$

$\angle PTR = 180 - 135$

$\angle PTR = 45^\circ$

$\angle STQ = \angle PTR$ (Vertically opp. Angles]

In $\triangle SQT$

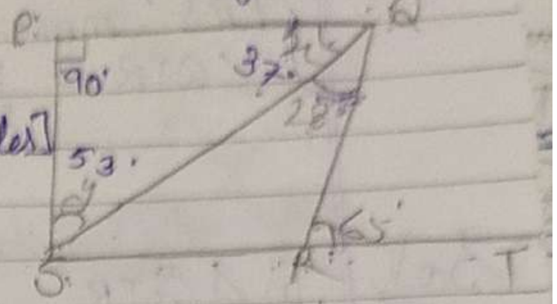
$\angle STQ + \angle TSQ + \angle SQT = 180^\circ$ [Angle sum proph. of $\triangle$]

$45^\circ + 75^\circ + \angle SQT = 180^\circ$

$\angle SQT = 180 - 120^\circ$

$\angle SQT = 60^\circ$

Qs. In fig. 6.43 if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.



Sol. $\angle PQR = \angle QRT$ [Alternate Interior Angles]

$x + 28^\circ = 65^\circ$

$x = 65 - 28$

$x = 37^\circ$

In ΔPQS

$\angle SPQ + x + y = 180^\circ$ [Angle Sum prop. of Δ]

$90 + 37 + y = 180^\circ$

$127 + y = 180^\circ$

$y = 180 - 127^\circ$

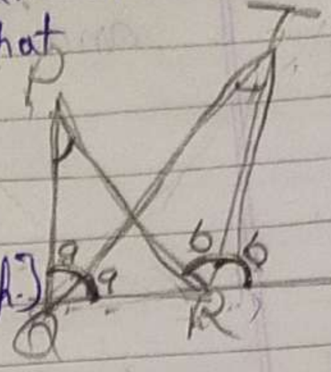
$y = 53^\circ$

$x = 37^\circ$

$y = 53^\circ$

Q6. In fig. 6.44, the side QR of ΔPQR is produced to point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that

$$\angle QTR = \frac{1}{2}\angle QPR.$$



Sol: In ΔQTR

$\angle b = \angle a + \angle QTR$ [Exterior Angle prop.]

$\angle b - \angle a = \angle QTR$ —— (1)

In ΔPQR

$2\angle b = \angle QPR + 2\angle a$

$2\angle b - 2\angle a = \angle QPR$

$2(\angle b - \angle a) = \angle QPR$

$2\angle QTR = \angle QPR$

$\angle QTR = \frac{1}{2}\angle QPR$ (H.P).